MINISTRY OF EDUCATION AND RESEARCH
TECHNICAL UNIVERSITY
OF CLUJ-NAPOCA, ROMANIA

Fundamental field Applied Sciences - Mathematics
Specialisation Mathematics

HABILITATION THESIS
– ABSTRACT –

MATHEMATICAL METHODS FOR COMPUTATIONAL COMPLEXITY REDUCTION OF NONLINEAR DYNAMICAL SYSTEMS BASED ON KOOPMAN OPERATOR THEORY

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– Cluj-Napoca –
2023

This Habilitation Thesis provides an overview of the candidate's academic achievements and research conducted over the course of 12 years, from 2011 to 2022, after obtaining her Ph.D., confirmed by the Ministerial Order no. 4387 of June 06, 2011.

Since finishing her doctoral studies, the candidate has continued to carry out constant research work in the domain of Applied Computational Mathematics, following two main scientific directions:

1. Hydrodynamic Stability Analysis of Swirling Flows (AMS Classification: 76E15, 76E07, 76F20, 65M70), and

2. Contributions to Koopman Operator Theory for Reduced-order Modelling of nonlinear dynamical systems (AMS Classification: 49M27, 49M41, 65N25, 65N30), respectively.

These research topics address two significant problems in the application of mathematical theory to the study of contemporary dynamical systems.

(a) A challenging area in contemporary mathematics study is the numerical approach to a nonlinear dynamical system. While linear dynamical systems may be completely defined in terms of a spectrum decomposition with eigenvalues and vectors, nonlinear dynamical systems are more difficult to characterize. The creation of a generic mathematical framework for the characterization of nonlinear dynamical systems is a major mathematical task for the twenty-first century. Many research groups around the world are currently interested in this topic [1,2]. The expansion of computer capability has resulted in the application of new mathematical theories to lower the computational complexity of the mathematical models that describe such systems [3].

(b) An even more current challenge appears in the case of the study of dynamic processes for which the identification of a precise mathematical model is difficult. The candidate approached the study of complex and realistic dynamics, such as those in meteorology [4] and epidemiology [5]. Usually, these systems were approached by appealing to simplifying assumptions, leading to significant errors when the precision of the numerical solution was critical. In the case of these increasingly complex systems, the candidate used a modern paradigm, which involves the identification of mathematical models directly from the data provided by observing the dynamics of the studied phenomena.

To face these challenges, the candidate developed advanced techniques and methods, such as reduced complexity mathematical modelling, and developed computational solutions that include deep learning and artificial intelligence. The advantage of using deep learning and artificial intelligence-assisted approaches in this scenario is their ability to identify the coefficients of reduced-order mathematical models with high precision.

Another significant advantage of these mathematical models, which have a significantly lower number of degrees of freedom than the initial models, is the identification of the modes that have the greatest influence on the solution, as well as the rapid reproduction of the system dynamics based on these dominant modes, using reasonable computing resources. Furthermore, the candidate proved that mathematical models of reduced order are characterized by a high-fidelity of the numerical solution.

The mathematical framework for deriving the reduced-order model [6] of a complex nonlinear dynamical system is provided by the Koopman operator theory [7]. Transposing the nonlinear dynamics into a reduced-order model, which is a linear model by construction, brings with it several benefits in terms of checking dichotomy properties [8, 9] and developing algorithms with short- and medium-term prediction capabilities [10].

The habilitation thesis is divided into two parts: Scientific, Professional and Academic Achievements and Plans for Evolution and Career Development, respectively. The research findings are discussed with particular references to candidate publications in journals and conference proceedings.

Chapter 1 is a technical presentation of the candidate's main scientific achievements and academic background, including an outline of the candidate's doctoral dissertation and postdoctoral research.

Chapter 2 focusses on scholarly scientific research and the corresponding contributions made between 2011 and 2014 in the areas of mathematical and computational stability analysis of swirling hydrodynamic systems. On the basis of a solid training in pure mathematics and obtaining knowledge of computational mathematics, the candidate led an interdisciplinary research work. The applications of this mathematical methods include stability study of confined vortices and nonparallel swirling flows by means of Linear Stability Analysis and Parabolized Stability Analysis, respectively.

The candidate's research on reduced-order modelling of nonlinear dynamical systems based on Koopman operator theory, conducted from 2015 to the present, is described in depth in Chapter 3. During 2015-2016, she intensified her interdisciplinary research on reduced-order modelling of complex swirling hydrodynamic systems governed by partial differential equations. She made significant contributions to this topic during her postdoctoral research project, where she pioneered the application of modal analysis for nonparallel swirling flows in hydraulic turbines.

Following that, the candidate conducted considerable research in the field of reduced-order modelling of nonlinear dynamical systems utilizing Koopman operator theory, working closely with experienced researchers from the United States, the United Kingdom, and China. Currently, the candidate works intensively in this research topic and obtains significant results for Navier-Stokes turbulent fluid models, Saint-Venant atmospheric models, Bateman-Burgers convection-diffusion models and epidemiological models, with multiple citations in journals, international books, and international doctoral theses.

Future objectives considered for scientific and academic advancement are presented in Chapter 4.

The bibliography contains a full inventory of the candidate's publications during 2011-2022 as well as other general references.

References

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