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TECHNICAL UNIVERSITY
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Mathematics

HABILITATION THESIS
– ABSTRACT –

MATHEMATICAL MODELS OF VISCOUS NANOFLUIDS AND POROUS MEDIA

Assoc. Prof. Dalia Sabina CÎMPEAN, PhD
Department of Mathematics
Technical University of Cluj-Napoca

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The present habilitation thesis contains results related to applied mathematics in the field of fluid mechanics. The most important aspects of the research work, which was done after I have finished Phd thesis till the present, are summarized in five chapters, such that, the first chapter contains contributions to fluid flows in channels, the second reports published results about clear Newtonian fluids and nanofluids in porous cavities, the third treats hybrid nanofluids in porous chambers, the fourth is related to Hyers-Ulam stability and the last discusses about future research directions.

The first chapter of the thesis is dedicated to a hot topic of fluid flows in channels with several practical applications. The reported problems are related to the mixed convection of fluid flows in inclined porous channels, continuing and completing one of the directions of the PhD thesis. The mixed convection is considered, and this means, both, free and forced convection rise into the system according to the imposed conditions of uniform heat flux at the walls and an uniform upward streamwise velocity distribution at the channel entrance.

The first main problem considers a channel filled with a fluid saturated porous media since the second is replacing the clear Newtonian fluid by a nanofluid to observe the behaviour of the flow in the new conditions. An analytical investigation is reported, and the results are obtained in terms of a mixed convection parameter and the Peclet number, as well as the inclination angle to the horizontal. It is found that, for a given inclination of the channel, the effect of increasing Peclet number is to reduce the inclination effect and to make the solution become more symmetric, i.e. more like the solution for a vertical channel. Conversely, for a givenvalue of Peclet and mixed convection parameter, the effect of decreasing the angle of inclination towards the horizontal is to increase the effect of this on the flow and temperature profiles. The minimization of entropy generation is also studied for the same conditions of the mixed convection Newtonian fluid flow into the channel. The calculated Bejan number reveals that, the influence of the heat transfer dominates the entropy generation mechanism.

Another important problem of this chapter is dedicated to fully developed mixed convection flow of a nanofluid through an inclined channel filled with a porous medium. The study, applied to nanofluids, was motivated by the practical applications of the results which can be used for optimal designs of the heat transfer equipment, due to the potential of this smart fluids for high rate of heat exchange incurring either little or no penalty in the pressure drop. The main conclusion is that the nanofluid greatly increase the heat transfer, even for small additions of nanoparticles in the base water fluid.

The last paragraph of the chapter is a study related to a hybrid nanofluid mixture which saturates the porous matrix into the channel. The results were focused to prevent the dissipation of energy by calculating the maximum thermal advantage at a minimum entropy generation in the system.

The second chapter presents problems about fluid flows in cavities. The chapter is split in two main parts, such that: the first part contains problems of nanofluid flows in wavy cavities with and without a porous medium included, while the second part approaches the behaviour of a clear Newtonian fluid and a nanofluid flow in porous cavities with sinusoidal temperature distribution at the walls.

In the first paragraph of the chapter, the free convection for a two-dimensional wavy cavity is presented in two problems by considering a cavity filled with a nanofluid based on water and nanoparticles and then, a cavity filled by a porous medium saturated with a nanofluid. An alternative nanofluid model of the previous approach (first chapter) is used here, such that, the work follow the nanofluid model proposed by Buongiorno which incorporates the effects of Brownian diffusion and thermophoresis. Important findings of these research refer to an intensification of convective flow and heat transfer when the Rayleigh number increases. At the same time, one can find more homogeneous distribution of nanoparticles for high values of the Rayleigh number. Also, an increase in Rayleigh leads to an increase in the time taken to attain the steady state. An increase of other parameter namely, the undulation number, leads to a weak intensification of convective flow and a reduction of Nusselt due to more essential cooling of the wavy troughs where the temperature gradient decreases. Also, it is found that the variations of the heater location illustrate a modification of the fluid flow and heat transfer. The upper position of the heater reflects the minimum heat transfer rate, while the position between the bottom part and the middle section characterizes an enhancement of heat transfer.

The second part of the chapter two presents' studies about natural convection in a square inclined porous cavity with sinusoidal temperature distribution on both side walls. Both cases, of clear fluid and nanofluid flow who saturate the porous matrix were studied and the results are published in papers. The research was motivated by several applications, such that, the study of heat transfer characteristics in glass melting tanks, where several burners placed above the glass tank create periodic temperature profiles on the surface of the glass melt. The problems were solved numerically, by using a second-order accurate finite-difference method. The results reveal that, by inclining the cavity, for the case of a fluid saturated porous media, the behaviour of the convective cells is completely changed comparing to no inclination of it. Also, the change of the isotherms is not so dramatic in this situation. The increase of inclination angle reflects a non-linear influence on average Nusselt number and fluid flow rate.

For the case of nanofluid flow, the parameters related to the inclination angle of the cavity and to the sinusoidal temperature at the walls, affect the convective pattern, transport flux and concentration of the nanoparticles inside the cavity. Such that, Nusselt number increases with the increase of the inclination angle of the cavity and with a decrease of the amplitude ratio of the sinusoidal temperature. For the Rayleigh-Darcy convection, the inclination of the cavity enhances the heat transport. Also, high thermophoresis parameter, moderate inclination of the cavity and low Lewis number show an important non-homogeneous distribution of the nanoparticles inside the cavity.

The entropy generation inside the cavity is also reported in a paper and is shortly presented in a paragraph of the second chapter. A free convection for a nanofluid saturated porous medium in an inclined cavity with sinusoidal temperature at the side walls and a heat source bellow is considered here. The main results reveal that, the existence of the irreversibility phenomena is affected by the conditions of the model and the values of the studied parameters. Such that, except for the horizontal case when the entropy generation is seen inside the cavity owing to the heat source bellow, as the cavity is inclined, the entropy generation is reported near the walls, influenced by the sinusoidal temperature and the heat source.

The third chapter is dedicated to hybrid nanofluids which are considered in a porous chamber under various condition. A representative part of the chapter contains a numerical investigation of a mixed convection in a porous trapezoidal chamber with a moving upper wall, saturated with a hybrid nanofluid, under the influence of vertical temperature difference. The trapezoidal geometry can be counted as a simulation of industrial applications that has a solid material motion inside the cavity, such as float glass manufacturing and continuous reheating furnaces. The performed analysis shows that trapezoidal cavities can be more efficient compared to square cavities. During analysis it was revealed that a raise of the Reynolds number reflects an augmentation of convective circulation and energy transport. Low values of Reynolds number illustrate a development of mixed convection and high values characterize the forced convection. A time needed for the formation of steady state rises with the Reynolds number. Also, an increase of the Darcy number reflects the heat transfer enhancement.

Another problem presented in the chapter three, computationally investigate the natural convection in a porous closed space saturated by a hybrid nanosuspension under the effects of an internal heat-conducting block and horizontal temperature difference. The impacts of the Darcy number, nanoadditives concentration and internal solid block material on the liquid motion structure and heat transference patterns were investigated. The reported results show that: a rise in Darcy number accelerates the convective circulation and energy transport; the addition of nano-sized solid particles suppresses the flow strength and thermal transference intensity; a growth in the internal solid block material's thermal conductivity intensifies the thermal transmission for $Da < 0.005$, but for $Da > 0.05$, an increase in the considered heat conductivity reduces the energy transference intensity.

The fourth chapter is dedicated to Hyers-Ulam stability. The motivation of this part is given by the fact that, many times, the mathematical modelling of real practical problems will be complicated and the study of the properties of the solutions for the given equations are of very much interest. So that, together with numerical and maybe experimental studies, a qualitative study will validate the results of a problem related to fluid mechanics. The chapter contains two important published results. The first problem is dedicated to the stability of the linear differential equation of higher order with constant coefficients in Aoki-Rassias sense. As a consequence, the Hyers-Ulam stability of the mentioned equation is obtained and a connection with dynamical systems perturbation is established. The second problem proves when the Euler equation characterizing homogeneous functions is stable in Hyers-Ulam sense.

The last part of the thesis is dedicated to ideas for future research and developments. First, the proposed mathematical models would be extended with other new problems involving viscous nanofluids, nanofluid saturated porous media and hybrid nanofluids. For example, the study the nanofluids and their hybrids by considering variable physical properties (depending by temperature, concentration, etc.) and the effect of the magnetic field on nanoparticles (driving particles problems using magnetophoresis and heat generation. Suitable CFD software will be used to solve problems concerning engineering or medicine applications.

Considering the future needs of society and promoting the development of sustainable and renewable energy, for long-term research, I will also focus on issues related to the optimization of energy systems and green thermal systems.

Regarding the teaching activity, I would like to use my experience and research to show relevant applications to the Special Mathematics courses and guide students and PhD students.