MINISTERUL EDUCAȚIEI

UNIVERSITATEA TEHNICĂ
DIN CLUJ-NAPOCA

Specialization: Mathematics

HABILITATION THESIS
-ABSTRACT-

Some contributions to the approximation of unbounded functions

Assoc. Prof. PhD. Adrian HOLHOŞ

Faculty of Automation and Computer Science

Technical University of Cluj-Napoca

Cluj-Napoca

2021

str. Memorandumului nr. 28, 400114 Cluj-Napoca, România
tel. +40-264-401200, fax +40-264-592055, secretariat tel. +40-264-202209, fax +40-264-202280
www.utcluj.ro

This habilitation thesis, entitled *Some contributions to the approximation of unbounded functions*, contains some results achieved by the author in the field of Approximation Theory after obtaining his Ph.D. degree in Mathematics at Babeş-Bolyai University. It represents a continuation of the study done in the period of the doctoral studies (2006-2010) under the supervision of Professor Ioan Gavrea concerning the positive and linear operators. The new results are presented in a more suitable framework, that of positive sublinear operators, which were introduced and studied in the last decade. In this way, old results related to positive linear operators and new results related to max-product type operators can be presented in a unified mode.

The thesis has two distinct parts. The first part contains some of the scientific achievements obtained by the author in the last four years. For the sake of brevity and unity, not all the mathematical results of the author are presented, but only the most significant ones. The second part of the thesis presents some future plans for the academic career.

The presentation of the results is didactic. We start in Chapter 1 with some important notions which are necessary for the presentation of the results contained in the thesis. We introduce the notion of weight space which represents the space of unbounded functions to be approximated. We give the definition and some properties of the positive sublinear operators. They represent a generalization of the positive linear operators and can be used for the approximation of functions. In the last section of the first chapter, we present a weighted modulus of continuity introduced by the author for the estimation of the error in the approximation of functions from weight spaces. In this general setting, a new result for the estimation of the error of approximation is presented.

In Chapter 2 we present three sequences of operators of max-product type. They are defined based on the classical operators of Szász-Mirakyan, Meyer-König and Zeller and a generalization of Favard operators. The results which we have obtained show the potential of the operators of this kind for approximating not only bounded functions but also unbounded functions. In all cases and for a large class of functions, the rate of approximation is better than the rate obtained in the case of the classical positive linear operators from which they originate. These max-product type operators were introduced and studied only in the last decade and the increasing number of research papers devoted to the study of these operators show a good future direction to be followed.

Chapter 3 contains results related to a particular type of positive linear operators generated by a differential equation called exponential type operators. For these operators, we can obtain the asymptotic behavior of the moments and some general Voronovskaya type results. Due to space requirements, we have limited our presentation to some general results, but we must mention that other general results can be given and many more particular cases.

Chapter 4 presents three examples of operators obtained from the classical ones by squaring their basis and using normalization. They are the squared Bernstein operators, the squared Szász-Mirakyan operators and the Baskakov operators. This type of operators obtained from classical exponential type operators is also of exponential type. So, some general results we have presented in Chapter 3 can be applied to these operators too. In this way, we obtain new results which are not yet published. We even have given an answer to a conjecture. For each of these three operators we give the generating differential equation, we obtain some representation for the first central moment and we study the asymptotic behaviour of the first central moment. These informations allow us to obtain some Voronovskaya type results in simultaneous approximation. For each of the three operators, we give an upper bound of the second central moment which is very important for the uniform approximation results.

The second part of the thesis contains Chapter 5, in which some of the academic future plans of the author are presented. I intend to continue my didactic and research activities at Technical University of Cluj-Napoca. As regards teaching to the undergraduate students, in this new context of online learning, I want to improve the methods used in education, to involve students in

an active-participatory way. I also plan to elaborate research proposals and supervise master and doctoral students.

I had so far a good collaboration with my colleague Daniela Roşca and we have written five papers together. I also have written a paper in collaboration with a team from the electrical engineering department and recently a joint paper with Vijay Gupta. These prove that the author can successfully work in teams, on the national and the international level. It is my intention to continue these collaborations in the future and to start new ones. In fact, I have just started a new collaboration with a research team from Harran University from Turkey. I hope it will be a fruitful one.

I plan to publish a monograph, which contains all the results obtained so far and new results which are expected to appear in the future projects. Some of these future projects I will describe briefly in the following.

I want to extend the study of exponential type operators in several directions. The generating differential equation of these operators can be given in a more general form to include other known types of positive linear operators. Not all of the positive linear operators are of exponential type. It is possible to extend the differential equation to a certain relation that will allow the inclusion in the general class of operators of other modifications of the classical operators (like that of Chlodovsky, or Kantorovich, or Durrmeyer)? What is the most general form of this equation which can be considered is a question for future research.

We have given some pointwise and asymptotic approximation results related to the exponential type operators. What other properties can be derived directly from the differential equation which they verify? Can we obtain uniform approximation results for this general class of operators? What can we say about the maximal class of functions which are approximated by a given sequence of exponential type operators? How can we determine it from the equation?

One class of exponential type operators which we began to study is that of operators constructed by using the square of the basis of some classical operators. What functions can be chosen instead of the power function such that the operators we obtain are of exponential type? What are the properties of these new operators? Do they have good approximation properties? This is another direction of future study.