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HABILITATION THESIS
– ABSTRACT –

RESULTS IN VARIATIONAL ANALYSIS AND APPROXIMATION THEORY

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This habilitation thesis is based on some of the most significant results published by the author after obtaining the PhD degree in Mathematics, in 2003, at Babeş-Bolyai University of Cluj-Napoca. The candidate joined the Department of Mathematics from the Technical University of Cluj-Napoca in the year 2000 and occupies currently a position of Associate Professor in this department.

The content of the thesis is structured in two parts. The first one presents the scientific, professional and academic achievements, and contains a selection of results that have been published in various papers, in two main research directions: Variational Analysis and Approximation Theory. Part II is focused on the plans for evolution and career development. The main part has four Chapters, thirteen Sections and References. In what follows, we mention some details about each of these chapters.

Chapter 1: *Equilibrium problems with trifunctions*; Sections: *Introduction, Existence theorems via duality for equilibrium problems with trifunctions, Calmness of the solution mapping*.

Equilibrium problems are studied intensively due to the large range of problems that they encompass and to their many applications. The results that compose this chapter were published in papers written together with Professor Joszéf Kolumbán from Babeş-Bolyai University (the PhD supervisor of the candidate). In the classical theory, the function that appears in the statement of the problem has two variables – a novelty of this work is the use of a function of three variables (called a trifunction). The presence of three variables allows the extension of pseudomonotonicity notions and more freedom in the conditions that are imposed.

Two main aspects are under investigation: the existence of a solution and the calmness of the solution mapping, when the problem depends on a parameter. Monotonicity and pseudomonotonicity are generalized from bifunctions to trifunctions, and sufficient conditions for the equivalence between primal and dual problems are given. Existence of solutions is proved using a version of the Ky Fan intersection lemma. The abstract results are then applied to mixed equilibrium problems, formulated with a sum of two functions that have different properties.

Calmness of a function is an important stability property since it gives a bound on the distance between perturbed solutions and unperturbed solutions. Most of the studies on calmness have focused on particular models such as optimization problems or variational inequalities. The results presented in the last section of Chapter 1 concern a more general equilibrium problem, subsequently particularized to a hemivariational-type inequality.

Chapter 2: *Variational relations problems;* Sections: *Introduction, Variational relations problems via fixed points of contraction mappings, Systems of quasi-variational relations, Factorization of quasi-variational relations systems, Calmness of the solution mapping of parametric variational relations problems.*

Variational relations problems appeared for the first time in 2008, in a paper of D.T. Luc, as a unified approach for variational inequalities, variational inclusions, optimization or equilibrium problems. In this chapter, fixed point theorems (Matkowski, Wegrzyk, Reich) for contraction mappings in complete metric spaces were used to prove existence results for variational relations and also for systems of variational relations problems. The abstract results were then applied to a multivalued equilibrium problem and to a Stampacchia quasi-variational inequality problem, that is a variational inequality where the constraint set depends too on the variable.

Systems of variational relations were also investigated by a factorization method, which focuses on the question: in which hypotheses the solvability of some partial problems implies the existence of a solution for the entire system? The results obtained aim to impose conditions that are weaker then closedness and convexity of relations. As examples, a Minty variational inequality and a constrained Nash equilibrium problem are presented.

In the case of problems depending on parameters, conditions for the calmness of the solution mapping were determined, in the last section of Chapter 2.

Chapter 3: *Generalized monotonicity in the context of variational inequalities;* Sections: *Introduction, Variational inequalities governed by pseudomonotone mappings, Variational-like inequalities governed by pseudomonotone mappings*.

This chapter contains studies on generalized monotonicity properties used in the more particular framework of variational and variational-like inequalities. More precisely, we discuss about three types of pseudo-monotonicity of set-valued mappings: algebraic, topological and a third one which, in a way, generalizes the first two. These notions are compared to each other, illustrated by examples, and considered as conditions that ensure the existence of solutions for variational inequalities.

Variational-like inequalities are also investigated. These extend the classical variational inequalities by replacing the difference "y-x" that appears in their formulation, by a more general function. In most of the existing papers in the literature, this function is supposed to be afine in the first variable. We considered the situation when it verifies a weaker assumption, namely it is of type ql in the first variable.

Chapter 4: *Approximation properties of Szász-Mirakyan type operators;* Sections: *Introduction, Generalized Szász-Mirakyan operators*.

Chapter 4 contains studies from a different research direction. While the previous chapters are essentially part of Nonlinear Analysis, the last one approaches problems from Approximation Theory, concerning linear operators - more precisely classical and modified Szsász-Myrakian operators. The results on this subject were obtained in joint work with Professor Ioan Raşa from Technical University of Cluj-Napoca and Professor Ali Aral from Kirikkale University, Turkiye.

A well-known linear positive operator, the Szsász-Myrakian operator acts on functions defined on the unbounded interval from 0 to infinity. It was introduced in 1950 and since then various extensions appeared in the work of many researchers. This chapter examines the main properties of a new approximation process based on such an operator: weighted uniform convergence, a Voronovskaya type theorem and shape preserving properties. Also, a difference between the classical Szsász-Myrakian operator and a modified one is estimated.

The results of the candidate's research work were published in 46 papers, among which 22 are in ISI journals. Some of the results were also disseminated during scientific events, by presentations in national and international conferences.

The total score, calculated from 18 papers published in journals with Relative Influence Score higher than 0.5, is 8.89.

The scientific impact, measured by the number of citations, is: - 248 citations, h-index=7 (Google scholar) - 155 citations, h-index=7 (Web of Science) and 31 citations in journals with the RIS higher than 0.5.

Other academic achievements of the candidate can be mentioned: served as a reviewer for several ISI ranked journals; was a member in commissions for evaluations of PhD theses and

occupations of acadmic positions; was involved as a team member in 3 research projects and in a teaching project; coordinated a research project obtained by national competition (*Shape sensitivity analysis for the solutions of some variational problems, AT - CNCSIS Grant*); participated in the organization of 7 editions of the *International Conference on Applied Mathematics and Computer Science*, organized by the Department of Mathematics-TUCN together with the association "Seminarul Theodor Angheluţă".

The didactic activity of the candidate includes teaching courses and practical works, in Romanian and English, for the following disciplines: Mathematical Analysis (differential and integral calculus, differential equations), Special mathematics in engineering (complex analysis, integral transforms), Linear algebra and Analytic geometry.

In Part II of the thesis, future plans for career improvement and development are presented, in what concerns both scientific and teaching activities. The candidate has the intention to pursue the research in (at least) several directions:

- Variational relation problems,
- Equilibrium problems (especially those defined by trifunctions, but not excluding the classical ones),
- Hyers-Ulam stability of integral and differential equations.

A few ideas meant to improve the management of academic career are also exposed (better dissemination of the results, increasing the international visibility, participating in competitions for grants).

As a member of the Department of Mathematics, the candidate wants to continue the teaching activity, using the over 20 years of experience. Briefly, some of the courses of action to be taken are:

- Keeping the contents of the lessons up-to-date and connected with applications,
- Improving communication skills,
- Writing and publishing new and revised materials dedicated to engineering students.